Solutions to JEE Advanced Booster Test - 4 | 2024 | Code A

[PHYSICS]

SECTION 1

1.(A) Let the acceleration of the lift be a_0 upwards

Let the acceleration of the blocks A and B relative to the lift be a downwards and a upwards respectively

From the force diagrams,

$$T - 6g = 6(a_0 - a)\,; \qquad T - 2g = 2(a_0 + a)$$

Solving, we get

$$T = 3(g + a_0)$$

Since the pulley is massless, the net force on it must be zero, hence the tension in the string connecting the pulley to the ceiling is $2T$

So, we have two conditions,

$$T < 60 \quad \text{and} \quad 2T < 90$$

Combining the two conditions, we get

$$t < 45$$

Therefore, $\quad 3(g + a_0) < 45$

Hence $\quad a_0 < 5$

2.(A) If the elongation of the spring is x, the displacement of each block is $\frac{x}{2}$ due to symmetry

Conserving energy,

Work done by the external forces = Change in spring energy + Change in kinetic energy

$$F_A\left(\frac{x}{2}\right) + F_B\left(\frac{x}{2}\right) = \frac{1}{2}kx^2 + 2\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 10(0.05) = \frac{1}{2}(200)(0.05)^2 + (1)v^2 \quad \Rightarrow \quad v = 0.5 \text{ m/s} = 50 \text{ cm/s}$$

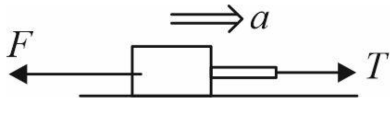
3.(D) $\frac{a_r}{a_t} = \tan\theta \quad \Rightarrow \quad \frac{\omega^2 r}{r\alpha} = \tan\theta$

$$\Rightarrow \quad \frac{(\alpha t)^2}{\alpha} = \tan\theta \quad \Rightarrow \quad t = \sqrt{\frac{\tan\theta}{\alpha}}$$

4.(B) From the FBD of entire system,

$$2F - F = (3 + 1 + 2)a \quad \Rightarrow \quad a = \frac{F}{6m} \text{ (towards right)}$$

Now, drawing the force diagram of the left half of the system,



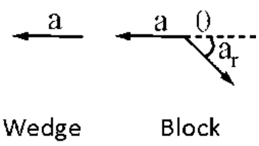
$$T - F = (3 + 0.5)a$$

$$\Rightarrow \quad T = F + (3.5)\left(\frac{F}{6m}\right); \qquad \boxed{T = \frac{19}{12}F}$$

5.(C) From constraint equation $a_r = 2a$

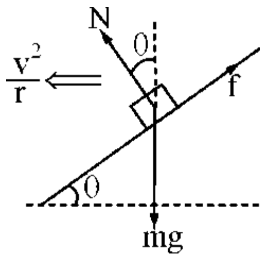
$\Rightarrow$ acceleration of block

$$= \sqrt{a^2 + (2a)^2 + 2(a)(2a)\,(-\cos\theta)}$$



$= a\sqrt{5-4\cos\theta}$

6.(AC) Assume friction is up the plane



Horizontal: $N\sin\theta - f\cos\theta = \frac{mv^2}{r}$

Vertical: $N\cos\theta + f\sin\theta = mg$

Solve to get $N = mg\cos\theta + \frac{mv^2}{r}\sin\theta$

$$f = \left| mg\sin\theta - \frac{mv^2}{r}\cos\theta \right|$$

7.(ABCD)

(A) Gain in KE = Loss in GPE

$$\frac{1}{2}m\left(v^2 - 2gL\right) = mgL\left(1-\cos\theta\right) \quad \Rightarrow \quad v = \sqrt{\frac{12gL}{5}}$$

(B) Gain in GPE = Loss in KE

$$mgL\cos\theta = \frac{1}{2}m\left(3gL - v^2\right) \quad \Rightarrow \quad v = \sqrt{\frac{7gL}{5}}$$

(C) Gain in GPE = Loss in KE

$$mgL\left(1+\cos\theta\right) = \frac{1}{2}m\left(\frac{24}{5}gL - v^2\right) \quad \Rightarrow \quad v = \sqrt{\frac{6gL}{5}}$$

Since this velocity is greater than $\sqrt{gL}$, the ball completes the circle

(D) Gain in GPE = Loss in KE

$$mgL\left(1+\cos\theta\right) = \frac{1}{2}m\left(6gL - v^2\right)$$

Since this velocity is greater than $\sqrt{gL}$, the ball completes the circle

$$\Rightarrow \quad v = \sqrt{\frac{12gL}{5}}$$

8.(ACD) The maximum separation between the blocks for which they can remain in equilibrium with the disc stationary is $\frac{5R}{3}$

The extension in the spring in this situation is $\frac{5R}{3} - R = \frac{2R}{3}$

So, $\mu mg = k\left(\frac{2R}{3}\right)$ … (i)

Now, when the disc is rotating, whether the spring is elongated or compressed, at the maximum allowable angular velocity for no slipping, friction acts radially inward on each block, since at higher angular velocities, the demand for radially inward force dominates

So, we have the following equilibrium equation:

$$\mu mg + k\left(\Delta l\right) = m\omega_M^2\left(\frac{R}{2} + \frac{\Delta l}{2}\right)$$

The above equation works for both positive and negative values of Δl

Using (i), $k\left(\frac{2R}{3}+\Delta l\right)=m\omega_M^2\left(\frac{R}{2}+\frac{\Delta l}{2}\right)$

9.(BC) (i) Taking x-axis along the direction of F_1,

$$W_1=\vec{F}_1\cdot\vec{s}=F_1 s_x$$

Here, s_x is the displacement along x, and hence, $s_x=\frac{F_x}{2m}t^2$

$$\Rightarrow \quad s_x=\frac{t^2}{2m}(F_1+F_2\cos\theta)$$

So, $W_1=\frac{t^2}{2m}F_1(F_1+F_2\cos\theta)$

(ii) Taking X-axis along F_2 and proceeding similarly to part (i), we get

$$W_2=\frac{t^2}{2m}F_2(F_2+F_1\cos\theta)$$

10.(ABD)

The block starts moving only if $\mu_1<\tan\theta$

Conserving energy,

Work done against friction = Loss in GPE

$$(\mu_1+\mu_2)(mg\cos\theta)(h/2\sin\theta)=mgh$$

So, $\mu_1+\mu_2=2\tan\theta$

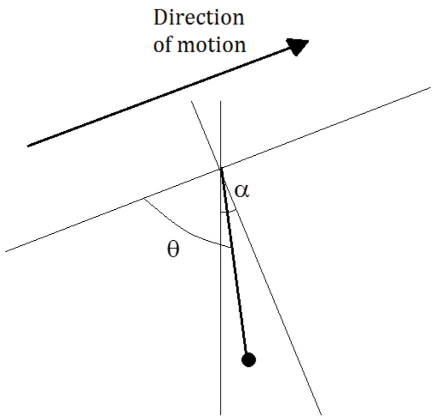
11.(B) The two situations of speeding up (acceleration in the same direction as velocity) and slowing down (acceleration opposite to velocity) are shown in the figure

They can be understood as follows:

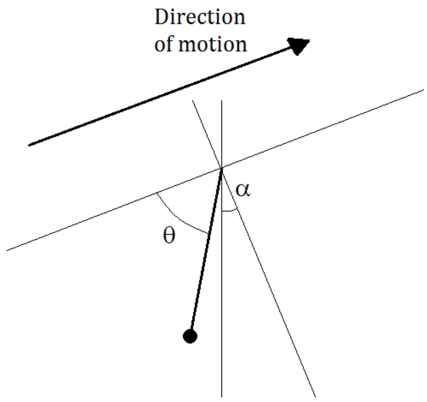
There are only two forces acting on the bob: gravity and tension from the string. Gravity is vertical, so in both cases, tension must provide the horizontal component of acceleration.

So, in the speeding up case, there must be a rightward component of tension, and hence in equilibrium, the string is on the left side of the vertical

And, in the slowing down case, there must be a leftward component of tension, and hence in equilibrium, the string is on the right of the vertical



$\theta>\frac{\pi}{2}-\alpha$, Plane slowing down



$\theta<\frac{\pi}{2}-\alpha$, Plane speeding up

12.(B) $\alpha = \cos^{-1}\left(\frac{24}{25}\right)$ and $\theta = \cos^{-1}\left(\frac{3}{5}\right)$

Therefore, $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha = \frac{7}{25}$

Since $\cos\theta > \cos\left(\frac{\pi}{2} - \alpha\right)$, $\theta < \frac{\pi}{2} - \alpha$

Hence, the plane is speeding up, and the situation looks like the second diagram above

Applying Newton's second law

(i) Parallel to the acceleration:

$T\cos\theta - mg\sin\alpha = ma$

(ii) Perpendicular to the acceleration:

$T\sin\theta = mg\cos\alpha$

Solving, we get $a = \left(\frac{\cos\alpha}{\tan\theta} - \sin\alpha\right)g$

Putting the values, $a = \left(\frac{\left(\frac{24}{25}\right)}{\left(\frac{4}{3}\right)} - \frac{7}{25}\right)g = \frac{11}{25}g$

SECTION 2

1.(32) Loss in KE = Work done against force of resistance

$$\frac{1}{2}m\left(v_0^2 - \left(\frac{v_0}{3}\right)^2\right) = F_{\text{metal}}t\,; \qquad \frac{1}{2}m\left(\left(\frac{v_0}{3}\right)^2 - \left(\frac{v_0}{6}\right)^2\right) = F_{\text{wood}}(3t)$$

Dividing the equations, we get , $\frac{F_{\text{metal}}}{F_{\text{wood}}} = 32$

2.(7.33)

Let the acceleration of the ball be a upwards. Then, the acceleration of the rod will be $2a$ downwards. So, w.r.t rod, ball moves up with an acceleration $3a$

$\therefore \quad t = \sqrt{\frac{2L}{3a}}$

Let, mass of rod = m, then, mass of ball $= \frac{3}{2}m$

$$T - \frac{3}{2}mg = \frac{3}{2}ma \qquad \dots \text{(i)}$$

$$mg - \frac{T}{2} = 2ma \quad \Rightarrow \quad 2mg - T = 4ma \qquad \dots \text{(ii)}$$

Adding (i) and (ii), we get

$$\frac{mg}{2} = \frac{11}{2}ma \quad \Rightarrow \quad a = \frac{g}{11}$$

Therefore, $t = \sqrt{\dfrac{2L}{3\left(\dfrac{g}{11}\right)}} = \sqrt{\dfrac{22L}{3g}}$

3.(6) $F \propto x^{1/2} \quad \Rightarrow \quad mv\dfrac{dv}{dx} \propto x^{1/2}$

$\Rightarrow \quad v^2 \propto x^{3/2} \quad \Rightarrow \quad u \propto x^{3/4}$

$\Rightarrow \quad \dfrac{dx}{x^{3/4}} \propto dt \quad \Rightarrow \quad x \propto t^4$

$\Rightarrow \quad v = \dfrac{dx}{dt} \propto t^3 \quad \Rightarrow \quad K.E. \propto t^6$

The total work done by the force until an instant is equal to the kinetic energy of the particle at that instant.

4.(3) We know that x_1 is given by:

$kx_1 = \mu mg \quad \Rightarrow \quad x_1 = \dfrac{\mu mg}{k}$

Now, let us assume that after the block is released, it moves and when it comes to rest for the first time, the spring is elongated by *x* (if the spring is actually **compressed**, the value of *x* will come out negative) Conserving energy,

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + \mu mg(x_0 + x)$$

$\Rightarrow \quad \dfrac{1}{2}k(x_0 - x) = \mu mg \quad \Rightarrow \quad x = x_0 - \dfrac{2\mu mg}{k}$

Now, for the block to not move again,

$x \le \dfrac{\mu mg}{k} \quad \Rightarrow \quad x_0 \le \dfrac{3\mu mg}{k}$

$\Rightarrow \quad x_2 = \dfrac{3\mu mg}{k}$

5.(5) $\bar{F} = 12xy^2\hat{i} + 6y\hat{j}$

$d\bar{r} = dx\hat{i} + dy\hat{j}$

$dW = \bar{F}.d\bar{r} = 12xy^2dx + ydy = 12x.x^4dx + 6ydy$

$W = \int_0^1 12x^5dx + \int_0^1 6ydy = 2x^6\Big|_0^1 + 3y^2\Big|_0^1 = 5J$

6.(7) We know that the minimum and maximum speed is achieved at the highest and lowest point respectively

So, by energy conservation,

$$\frac{1}{2}mv_{MAX}^2 - \frac{1}{2}mv_{MIN}^2 = mg\left(2R\right)$$

Also, we are given that $\dfrac{v_{MAX}}{v_{MIN}} = \sqrt{3}$

Solving, we get $v_{MAX} = \sqrt{6gR}$ and $v_{MIN} = \sqrt{2gR}$

So, at the lowest point,

$$N - mg = \frac{mv_{MAX}^2}{R} \qquad \Rightarrow \qquad N = 7mg$$

[CHEMISTRY]

SECTION 1

1.(C) $[N_2F]^+$ $[SbF_6]^-$
$\downarrow$ Linear, $\downarrow$ Octahedral

2.(A) $$\frac{r_{\text{ozonised mixture}}}{r_{O_2}} = \sqrt{\frac{32}{M_{mix}}} = 0.95$$

$$M_{mix} = \frac{32}{0.95 \times 0.95} = 35.46\,\text{g/mol}$$

Let x be the mole fraction of O_3 and 1 – x be the mole fraction of O_2.

$$x \times 48 + (1 - x)32 = 35.46$$

$$48x - 32x = 35.46 - 32$$

$$16x = 3.46 \quad \Rightarrow \quad x = 0.2162$$

% of $O_3 = x \times 100 = 0.2162 \times 100 = 21.62\%$

3.(D) AX_3 can be $\ddot{N}H_3$, BF_3, $\ddot{X}eO_3$, SO_3 etc.
So most appropriate option is (D)

4.(C) Beyond critical temperature, a real gas can't be liquified.
At critical temperature, liquid and gas phase are indistinguishable. Ideal gases can't be liquify at any value of temperature and pressure.

5.(A) **(A)** $\underset{(108°)}{N(CH_3)_3} < \underset{\approx 120°}{N(SiH_3)_3}$ **(B)** $\underset{(109°)}{NH_4^+} > \underset{(107°)}{NH_3}$

(C) $BF_3 = BCl_3$ (120°) **(D)** $SiCl_4 = CCl_4$ (109°)

6.(ABD)

(A) $$P = \frac{2 \times 0.0821 \times 300}{8.21} = 6\,\text{atm}$$

(B) For 1 mole

$$P(V_m - b) = RT$$

$$\Rightarrow \quad PV_m - Pb = RT \quad \Rightarrow \quad PV_m = RT + Pb$$

Or $$\frac{PV_m}{RT} = Z_m = 1 + \left(\frac{Pb}{RT}\right); \; Z_m > 1$$

(C) $$\left(P + \frac{an^2}{V^2}\right)V = nRT \quad ; \qquad P = \frac{nRT}{V} - \frac{an^2}{V^2}$$

Hence, would have pressure lesser than 6 atm.

(D) $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{8.21}{300} = \frac{V_2}{600} \quad \because \quad V_2 = 16.42\,\text{L}$$

7.(ABCD)

8.(ACD)

9.(ABD)

From bulb (I)

PV = nRT

$$V=\frac{nRT}{P}=\frac{0.3\times0.082\times400}{1}; \qquad V=0.3\times0.082\times400\,L$$

Calculation of number of moles of H_2 diffused in bulb (II)

$$\frac{r_{H_2}}{r_{O_2}}=\frac{n_{H_2}\text{ diffused}}{n_{O_2}\text{ diffused}}=\frac{P_{H_2}}{\sqrt{M_{H_2}\cdot T_{H_2}}}\frac{\sqrt{M_{O_2}\cdot T_{O_2}}}{P_{O_2}}$$

$$\frac{n_{H_2}\text{ diffused}}{n_{O_2}\text{ diffused}}=\frac{n_{H_2}}{0.1}=\frac{1}{\sqrt{2\times100}}\times\frac{\sqrt{32\times350}}{2}=\frac{\sqrt{14}}{2}=\frac{3.7}{2}=1.85$$

$\therefore$ Number of moles of H_2 diffused in bulb (II) = 0.185

For bulb (II)

Volume $=3V=3\times0.3\times0.082\times400\,L$

Number of moles of O_2 initially in bulb $(III)=\frac{PV}{RT}=\frac{2\times3\times0.3\times0.082\times400}{0.082\times350}=2.05$

Number of moles of O_2 remaining after diffusion $=2.05-0.1=1.95$

$\because \quad \frac{P_{initial}}{P_{after\ diffusion}}=\frac{2.05}{1.95}; \qquad P_{after\ diffusion}=P_{initial}\times\frac{1.95}{2.05}=2\times\frac{1.95}{2.05}=1.9\,atm$

Number of moles of H_2 remaining in bulb $(I)=0.3-0.185=0.115$

At constant T & V, $P\propto n$

$\because \quad \frac{P_{after\ diffusion}}{P_{initial}}=\frac{\text{number of moles after diffusion}}{\text{number of moles initial}}$

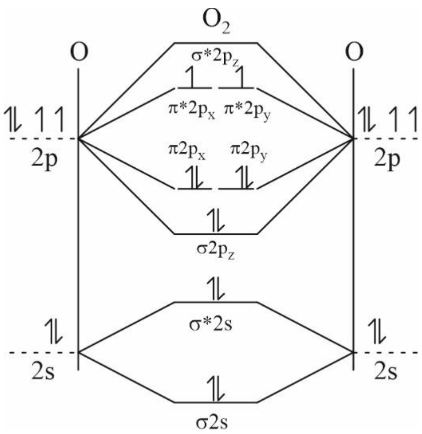
$\because$ Pressure in bulb (I) after diffusion $=1\times\frac{0.115}{0.3}=0.38\,atm$

10.(BCD)

Statements (B), (C) and (D) are correct according to M.O.T. Statement (A) is not correct as NO has 1 unpaired electron while O_2 has unpaired electrons.

Bond order :	O_2^+	O_2	N_2	NO^-	H_2^+	H_2^-
	2.5	2.0	3.0	2.0	0.5	0.5

11.(B)



As energy of valence e^- present in HOMO of O_2 is higher than the e^- present in valence orbital of O atom.

$\because$ Ionization energy of O_2 < O.

12.(C)

Orbital	**Number of nodal planes**
σ^*_{p-p}	1
π_{d-d}	1
π^*_{p-p}	2
π_{d-p}	1

SECTION 2

1.(4) For 1 mole of real gas,

$$\left(P+\frac{a}{V_m^2}\right)(V_m-b)=RT \qquad \ldots\ldots \text{(i)}$$

At very high pressure, equation (i) reduces to $Z=1+\frac{Pb}{RT}$

Slope $=\frac{b}{RT}=\frac{1}{1.4} \quad \Rightarrow \quad b=\frac{R\times T}{1.4}=\frac{22.4}{273}\times\frac{273}{1.4}$

$b=4\times$ volume of one mole of gas particles

$\because$ Volume of 1 mole of gas particles $=\frac{b}{4}=\frac{22.4}{1.4\times 4}=4$

2.(3) NH_4I, $NaBF_4$ and KI_3 contain all three types of bonds.

3.(4) $PV=nRT$

$$n=\frac{PV}{RT}=\frac{2.46\times 10}{0.0821\times 300}=1$$

Let moles of $C_xH_8=$ a moles

$\because$ Moles of $C_xH_{10}=(1-a)$ moles

Mass of carbon in $C_xH_8=a\times 12x$

Mass of carbon in $C_xH_{10}=(1-a)\times 12x$

Total mass of carbon $=12ax+(1-a)12x=12x$

% of carbon by mass $=\frac{\text{mass of carbon}}{\text{total mass}}\times 100$

$\because \quad 84.5=\frac{12x}{56.8}\times 100 \quad \because \quad 12x=\frac{84.5\times 56.8}{100}$

$\Rightarrow \quad x=\frac{84.5\times 56.8}{100\times 12} \quad \Rightarrow \quad x=4$

4.(3) According to ideal gas equation, at constant temperature.

$P_1V_1=P_2V_2$

$10\times 10=P_2\times 2.5 \quad \Rightarrow \quad P_2=40$

So, depth of water $=40-10=30\,m=3000\,cm=3\times 10^3\,cm$

5.(8) PCl_3F_2 $k_1 = 3$

PF_5 $k_2 = 0$

PCl_5 $k_3 = 5$

$\because \quad k_1 + k_2 + k_3 = 3 + 0 + 5 = 8$

6.(25) At Boyle's temperature, Z = 1

$\because \quad 1 = 1 + 0.48\,P - \frac{168}{T_B}P \quad \because \quad 0.48\,P = \frac{168\,P}{T_B}$

$\Rightarrow \quad T_B = \frac{168}{0.48} = 350\,K \quad \Rightarrow \quad \frac{5T_B}{7} = \frac{5}{70} \times 350 = 25\,K$

[MATHEMATICS]

SECTION 1

1.(B) $a_n = 16\left(\frac{1}{4}\right)^{n-1}$; $\therefore$ $P_n = \prod_{k=1}^{n} 16\left(\frac{1}{4}\right)^{k-1}$

$= 16^n \prod_{k=1}^{n}\left(\frac{1}{4}\right)^{k-1} = 16^n\left(\frac{1}{4}\right)^{0+1+2+....+(n-1)}$

$= 16^n\left(\frac{1}{4}\right)^{\sum_{i=0}^{n-1} i} = 16^n\left(\frac{1}{4}\right)^{\frac{n(n-1)}{2}}$

$= 2^{4n}.2^{-n(n-1)} = 2^{n(5-n)}$ $\Rightarrow$ $P_n^{1/n} = 2^{5-n}$

$\therefore$ $\sum_{n=1}^{\infty} P_n^{1/n} = \sum_{n=1}^{\infty} 2^{5-n} = 2^5\sum_{n=1}^{\infty} 2^{-n} = 32$

2.(C) $\because$ $2b = a + a^2$ and $(a^2)^2 = ab \Rightarrow a^3 = b$

$\Rightarrow$ $2a^3 = a^2 + a \Rightarrow a(2a^2 - a - 1) = 0$ $\Rightarrow$ $a(2a+1)(a-1) = 0$

$\therefore$ $a < 0$ $\therefore$ $a = \frac{-1}{2}$, $b = \frac{\frac{-1}{2} + \frac{1}{4}}{2} = \frac{-1}{8}$

$\therefore$ G.P. is $\frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, ...$ $\therefore$ Sum $= \frac{\frac{-1}{2}}{1 + \frac{1}{2}} = \frac{-1}{3}$

3.(C) $a = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ...\right)$

$= b + \frac{1}{4}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...\right)$ $\Rightarrow$ $a = b + \frac{a}{4} \Rightarrow \frac{3a}{4} = b$

4.(D) $\bar{z} = -i\bar{\omega} \Rightarrow \frac{z}{\omega} = i \Rightarrow \arg z - \arg\omega = \frac{\pi}{2}$ … (1)

$\arg(z\omega) = \pi \Rightarrow \arg z + \arg\omega = \pi$ … (2)

From (1) and (2), $\arg z = \frac{\pi + \pi/2}{2} = \frac{3\pi}{4}$

5.(C) $z^3 + 2z^2 + 2z + 1 = 0$ $\Rightarrow$ $(z+1)(z^2 + z + 1) = 0$

$\Rightarrow$ $z = -1, \omega, \omega^2$

For $z = -1$

$(-1)^{1985} + (-1)^{100} + 1 \neq 0$

For $z = \omega$

$(\omega)^{1985} + (\omega)^{100} + 1 = \omega^2 + \omega + 1 = 0$

For $z = \omega^2$

$(\omega^2)^{1985} + (\omega^2)^{100} + 1 = \omega + \omega^2 + 1 = 0$

$\therefore$ $z = \omega, \omega^2$ satisfies both the equations.

6.(ABCD)

Using $A.M. \geq G.M.$

$$\frac{\sum a_i}{16} \geq \left(a_1 a_2 .....a_{16}\right)^{1/16}$$

$\therefore \quad a_1 a_2 .....a_{16} \leq \left(\frac{a_1 + a_2 + ...+ a_{16}}{16}\right)^{16} \leq \left(\frac{392}{16}\right)^{16} \leq \left(\frac{49}{2}\right)^{16}$

$\Rightarrow \quad S = 49$ and $W = 2$

$$\frac{6}{2}\left(2a + 5.3d\right) = 147$$

$2a + 15d = 49$; $\qquad 2a + 15d = M = 49$

Also, $\frac{4}{2}\left(2a + 3.5d\right) = N$; $\quad \left(2a + 15d\right)2 = N \Rightarrow N = 98$

7.(ABC)

Suppose, z, iz, $-z$ and $-iz$ are represented by A, B, C and D in the complex plane. We have mid-point of AC is $\frac{1}{2}\left(z + (-z)\right) = 0.$ and mid-point of BD is $\frac{1}{2}\left(iz + (-iz)\right) = 0.$

Thus, the diagonals bisect each other.

Also, $AC = |-z - z| = 2|z|$ and $BD = |-iz - iz| = 2|-i|\,|z| = 2|z|$

$\therefore \quad AC = BD$

Finally, $AB = |iz - z| = |i - 1||z| = \sqrt{2}\,|z|$ and $BC = |-z - iz| = |-1 - i||z| = \sqrt{2}\,|z|$

Hence, ABCD is a square.

8.(BC) $|z_1 + z_2|^2 = \left(z_1 + z_2\right)\left(\bar{z}_1 + \bar{z}_2\right) = |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2$

We have $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0 \Rightarrow z_1\bar{z}_2 = -\bar{z}_1 z_2 \Rightarrow \frac{z_1}{z_2} = -\left(\frac{\bar{z}_1}{\bar{z}_2}\right)$

So, $\frac{z_1}{z_2}$ is purely imaginary.

9.(AB) We have $\left|\frac{z_1 + i\,\frac{z_1 + z_2}{2}}{z_1 - i\left(\frac{z_1 + z_2}{2}\right)}\right| = 1 \Rightarrow \left|\frac{\frac{2z_1}{z_1 + z_2} + i}{\frac{2z_1}{z_1 + z_2} - i}\right| = 1 \Rightarrow \frac{2z_1}{z_1 + z_2}$ is purely real

$\Rightarrow \quad \frac{z_1 + z_2}{z_1}$ is purely real $\Rightarrow \quad \frac{z_2}{z_1}$ is purely real.

10.(AB)

$$\frac{\underbrace{(x + x + ...x)}_{x \text{ times}} + \underbrace{y + y + ...y}_{y \text{ times}} + \underbrace{z + z + ...z}_{z \text{ times}}}{(x + y + z)} \geq \left(x^x y^y z^z\right)^{\frac{1}{x+y+z}}$$

$$\geq \frac{x + y + z}{\underbrace{\left(\frac{1}{x} + \frac{1}{x} + ...\frac{1}{x}\right)}_{x \text{ times}} + \underbrace{\left(\frac{1}{y} + \frac{1}{y} + ...\frac{1}{y}\right)}_{y \text{ times}} + \underbrace{\left(\frac{1}{z} + \frac{1}{z} + ...\frac{1}{z}\right)}_{z \text{ times}}}$$

11.(C) **12.(A)**

$$\frac{z+1}{z}=(-1)^{1/7}=e^{\left(\frac{i2r\pi+\pi}{7}\right)},\quad r=1,2,3,....7$$

$$\Rightarrow \quad z=\frac{1}{e^{i\theta}-1},\text{ where } \theta=\frac{2r\pi+\pi}{7}$$

$$=\frac{1}{(\cos\theta-1)+i\sin\theta}=\frac{1}{(2i\sin\theta/2)(i\sin\theta/2+\cos\theta/2)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{2i\sin\frac{\theta}{2}}$$

$$\Rightarrow \quad Z=\left(-\frac{1}{2}\right)-i\left(\frac{\cot\frac{\theta}{2}}{2}\right) \quad \Rightarrow \quad Z_r=-\frac{1}{2}-\frac{i}{2}\cot\left(\frac{2r\pi+\pi}{14}\right)$$

$$Z_r=-\frac{1}{2}-\frac{i}{2}\cot\left(\frac{2r\pi+\pi}{14}\right) \quad \Rightarrow \quad \sum_{r=1}^{7}\text{Re}(Z_r)=\frac{-7}{2}$$

$$\sum_{r=1}^{7}I_m(Z_r)=\frac{1}{2}\left[\cot\frac{3\pi}{14}+\cot\frac{5\pi}{14}+\cot\frac{7\pi}{14}+\cot\frac{9\pi}{14}+\cot\frac{11\pi}{14}+\cot\frac{13\pi}{14}+\cot\frac{15\pi}{14}\right]=0$$

SECTION 2

1.(1) $$\text{Sum}=\frac{1}{3}\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+....\right)$$

$$=\frac{2}{3}\left(\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}+....\right)=\frac{2}{3}\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+....\right)=\frac{1}{3}$$

2.(5) $$z=e^{\frac{i2r_1\pi}{15}}=e^{\frac{i2r_2\pi}{25}} \quad \Rightarrow \quad \frac{r_1}{15}=\frac{r_2}{25}\Rightarrow 5r_1=3r_2$$

r_1	0	3	6	9	12
r_2	0	5	10	15	20

3.(6) $$|9z_1z_2+4z_1z_3+z_3z_2|=|z_1z_2z_3|\left|\frac{9}{z_3}+\frac{4}{z_2}+\frac{1}{z_1}\right|$$

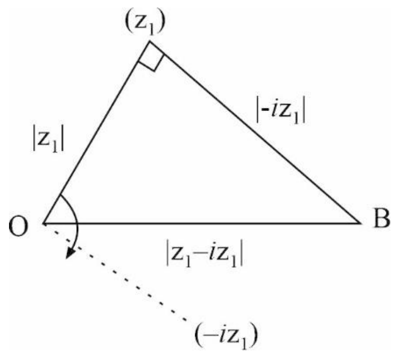
$$|z_1z_2z_3||\overline{z}_3+\overline{z}_2+\overline{z}_1|=|z_1||z_2||z_3||z_1+z_2+z_3|=6$$

4.(3) Let $z_1=2z-z^2$

Area of $\Delta AOB=\frac{1}{2}|z_1||-i\,z_1|=\frac{1}{2}|z_1|^2$

$$=\frac{1}{2}|2z-z^2|^2=\frac{1}{2}|z(2-z)|^2$$

$$=\frac{1}{2}|z|^2|2-z|^2=16\Rightarrow(4+b^2)b^2=32$$



$\Rightarrow \quad b^2=4$

$\therefore \quad |z|^2=a^2+b^2=8 \Rightarrow |z|=2\sqrt{2}$

5.(22) $\frac{a_1+2}{2}=\sqrt{2a_1}\Rightarrow(a_1-2)^2=0\Rightarrow a_1=2$

$(a_n+2)^2=8s_n$

$(a_{n-1}+2)^2=8s_{n-1}$

$\Rightarrow \quad (a_n+2)^2-(a_{n-1}+2)^2=8a_n\Rightarrow(a_n-a_{n-1})(a_n+a_{n-1}+4)=8a_n$

$\Rightarrow \quad a_n^2-a_{n-1}^2=4(a_n+a_{n-1})\Rightarrow a_n-a_{n-1}=4\Rightarrow a_1, a_2, a_3, ...$ are in AP

$\therefore \quad a_6=a_1+5(4)=2+5\times4=22$

6.(80) Let first term of G.P. be a and ratio be r.

$\Rightarrow \quad a+ar+ar^2=70$ and $10ar=4a+4ar^2$

$\Rightarrow \quad a=40,\ r=\frac{1}{2}$

$$S=\frac{a}{1-r}=\frac{40}{1-\frac{1}{2}}=80$$